



A computational methodology to calculate the required power in disc crushers

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Abstract

This study aims to contribute to the estimation of power consumption in a disintegration process in disc crushers (fixed and mobile). The study covers the dynamic analysis of forces acting on the particles and the mobile disc. A detailed analysis of the resultant force on the particles was performed. Finally, the consumed power is calculated with the forces acting on the mobile disc. The calculated power is a key aspect in the design of disc crusher machines.

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Keywords: Power estimation; Mill disc; Disc crushers; Disintegration process

1. Introduction

Disc crushers are widely used in the agricultural, wood, mining and chemical industries [1]. For example, the studies on the effects of milling on oil quality [2], the effects of different mechanical crushers in the process of olive paste [3], the studies on the influence of physical properties of seeds on shelling performance using a disc mill [4] are recent and important studies that suggest the need to fully understand the physics and mechanics in the design and performance of new types of disc crushers.

The relationship between energy consumption and product size in a disintegration process is the fundamental pillar of the theory of the disintegration process [5]. For that reason, it is important to obtain the power consumption, which is related to energy consumption in a time interval. Consequently, the estimation of the required power for disc crushers is essential in a fragmentation process.

Although there are theories that give an approximation of the energy consumption in a process of disintegration, currently, there is no a satisfactory and general one. Among them, the theory of Von Rittinger (1867), who believed that the required energy in the milling process must be related to the new surface produced during this operation. Another theory is the theory of Kick, who believes that the required energy for size reduction of two particles is proportional to the reduction in volume or mass of these particles [6]. It is also important to mention the theory called Bond Law, which states that the work required in a process of disintegration is proportional to the square root of the diameter of the particles produced [7]. These three proposed theories can be expressed in a single equation known as Walker Equation, which states that the energy required for size reduction of a material is proportional to the amplified n times size [8].

Another method describing a process of disintegration is "The Population Balance Equation", which is a mathematical description of how the distribution of particles according to their size changes depending on time [9]. It is worth mentioning that there are several commercial softwares which allow the estimation of the consumed power in a disintegration process. These softwares use numerical method which are based on the method of discrete elements [10], for example the software called Rocky.

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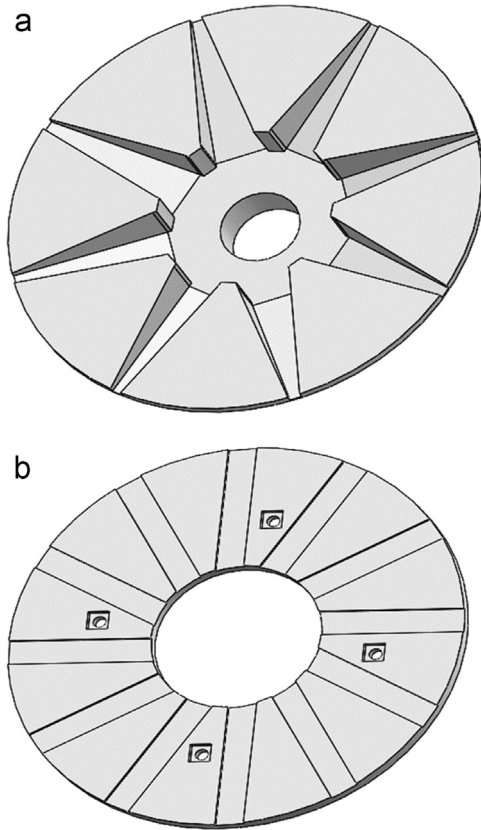


Fig. 1. Discs crushers, (a) mobile disc, (b) fixed disc.

In this study, a methodology to calculate the required power in a disintegration process is presented. This analysis considers relative movement between particles and disc crushers, which includes coriolis and centripetal forces. Worth mentioning that the disc crushers were designed by the authors of this paper.

2. Methodology

A disintegration process is closely related to the size reduction [11]. For this reason, it is necessary to analyze the size reduction through the disc channel.

2.1. Technical description of fixed and mobile disc

It is worth to mention that the discs were designed to mill sweet corn. Fig. 1 shows the discs used in this paper to evaluate the calculation methodology of required power during milling. The disc crusher of quadrangular channel is utilized in further discussions, see Fig. 1a. Its geometry was design to mill properly sweet corns. As it can be seen, the section is reduced as it approaches to the boundary of the disc, see Fig. 1. It is assumed that sweet corns are spherical and compact and as son them enter into the disc crusher the milling process starts. The number of channels of the disc depends of the volume of the first spherical body and they are used to guide the corn through all the milling process until the final process. The fixed disc has special channels, which have the function of cutting the corn while it is rotating in the channel of the mobile disc.

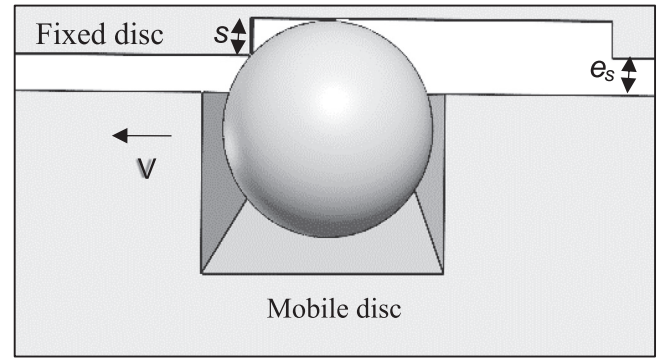


Fig. 2. Cutting process of the particle, “s” is the cut depth and “e_s” is the distance between the flat faces of the discs.

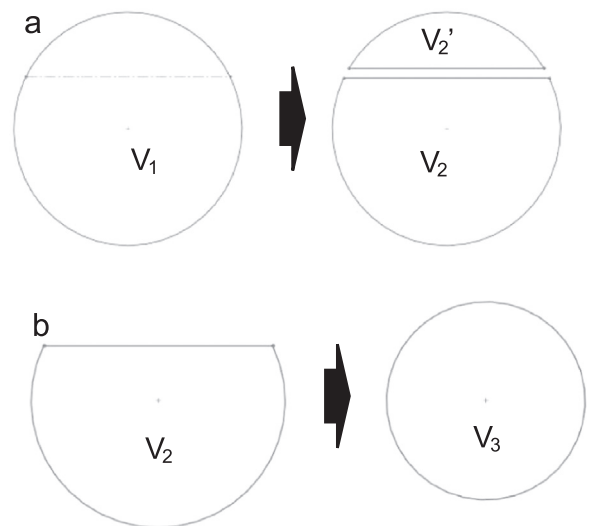


Fig. 3. The process of disintegration of the particle, (a) generation of new elements, (b) generation of the new sphere of volume V₃.

2.2. Methodology of calculation

It is considered that the product to be fragmented is a sphere, because the sphere is more compact and it has higher shear strength than other solids, consequently the method of estimation of power is conservative. The forces that normally act in a process of fragmentation are: compression, shear, impact and abrasion [12]. The present disc is designed to cut as shown in Fig. 2.

In each cut, the volume of the sphere V₁ becomes two elements V₂ and V₂'. The element V₂' is, in terms of volume, much lower than the element V₂; this is due to the fixed disc geometry, which ultimately determines the cutting depth. For purposes of calculation, it is assumed that the solid of volume V₂ becomes a new sphere of volume V₃, see Fig. 3.

To evaluate the power consumption in the disintegration process, it is necessary to evaluate the power consumption in each channel of the mobile disc, see Fig. 4.

To obtain the power on each disc channel, the forces \vec{N}_1 and \vec{N}_2 , acting on the mobile disc, have to be calculated, see Fig. 5.

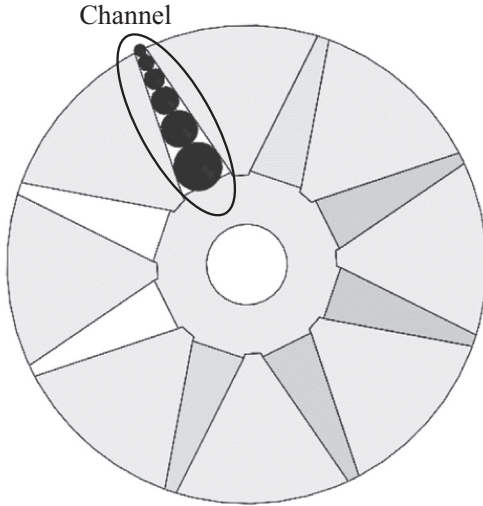


Fig. 4. Distribution of spheres in a disc channel.

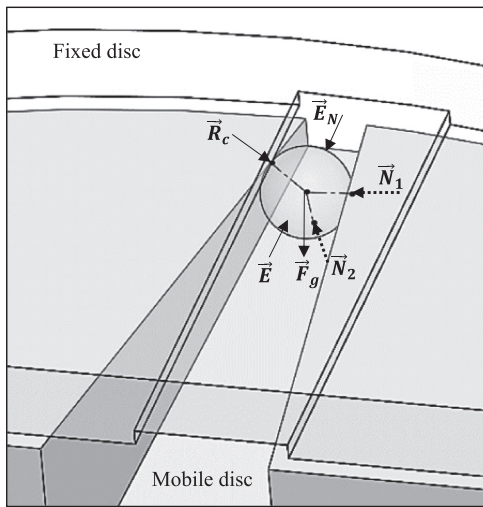


Fig. 5. Reaction forces on the sphere.

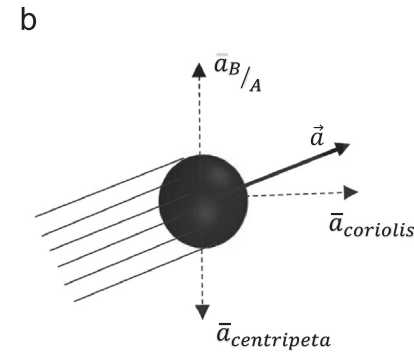
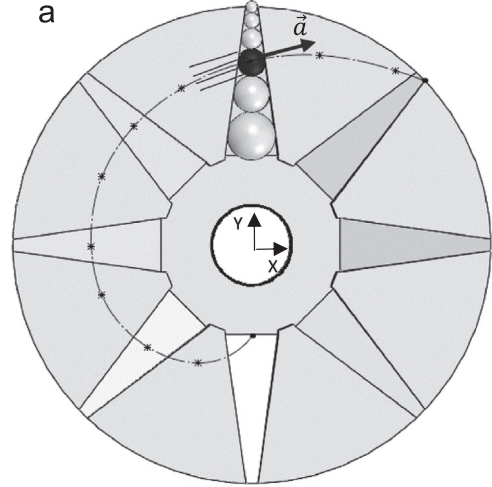


Fig. 6. Components of the resultant acceleration, (a) particle trajectory, (b) components of the resulting acceleration of the particle.

where the centripetal acceleration $\bar{a}_{centripeta}$ is equal to $\bar{\omega}x(\bar{\omega}x\bar{r}_{B/A})$. The Coriolis acceleration $\bar{a}_{coriolis}$ is equal to $\bar{\omega}x(\bar{V}_{B/A})_{xyz}$, and $(\bar{a}_{B/A})_{xyz}$ is the relative acceleration of the sphere with respect to the disc, see Fig. 6; $\bar{r}_{B/A}$ is the relative position of the sphere with respect to the mobile disc, $\bar{V}_{B/A}$ is the relative velocity of the sphere with respect to the mobile disc, $\bar{\omega}$ is the disc angular velocity.

$\bar{r}_{B/A}$, is determined according to Eq. (4).

$$\bar{r}_{B/A} = \frac{D_B}{2} - ctg\alpha(R + s + e_s), \quad (4)$$

where D_B is the outer diameter of the disc, R is the radius of the sphere, α is half the coning angle of the mobile disc, see Fig. 7.

The spheres pass from cut to cut process in an interval of time t and it is calculated according to Eq. (5), see Fig. 8.

$$t = \frac{2\pi}{N\omega}, \quad (5)$$

where N is the number of cutting channels of the fixed disc and ω is the angular speed at which rotates the mobile disc.

In each section the sphere reduces its size, consequently, the sphere moves radially through the disc channel. The sphere may occupy several possible radial positions, which in time draws a curved path as shown in Fig. 8. In the position 1' the sphere occupies its maximum radial position and in the position 2' occupies any radial position before position 1'. In Fig. 8, the cutting section of position 1' presents a greater cross

The unknown forces are: \vec{E} , \vec{N}_1 , \vec{N}_2 y \vec{E}_N .

$$\vec{N}_1 + \vec{N}_2 + \vec{R}_c + \vec{E} + \vec{E}_N + \vec{F}_g = \vec{F}_R, \quad (1)$$

where \vec{R}_c is the reaction force of the fixed disc on the sphere, \vec{E} is the thrust force, \vec{E}_N is the reaction force due to the action of the thrust force on the continuous sphere. \vec{N}_1 and \vec{N}_2 are normal forces on the sphere. \vec{F}_g is the gravity force.

When the last sphere is about to leave the disc, the force \vec{E}_N is zero; because there is no sphere after it, so the number of unknowns in Eq. (1) is reduced to three. These unknowns can be found from the resultant force on the sphere and the force \vec{R}_c according to Eq. (2).

$$\vec{N}_1 + \vec{N}_2 + \vec{R}_c + \vec{E} = \vec{F}_R. \quad (2)$$

The resulting acceleration is calculated to obtain the resultant force, see Eq. (3).

$$\bar{a}_{TOTAL} = \bar{a}_{centripeta} + \bar{a}_{coriolis} + (\bar{a}_{B/A})_{xyz}, \quad (3)$$

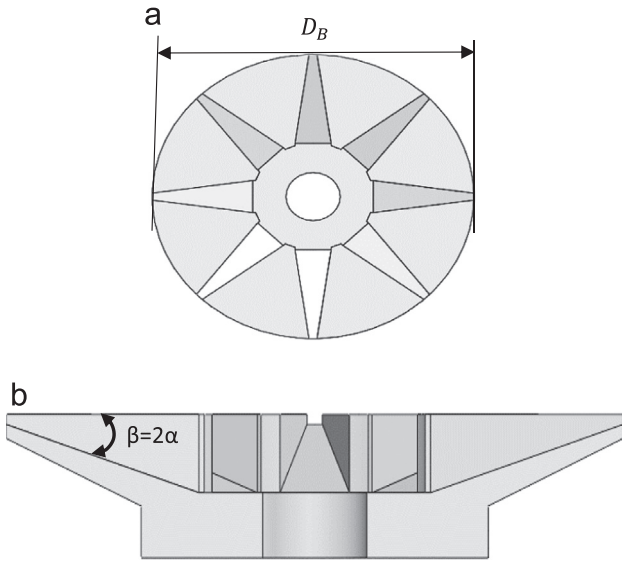


Fig. 7. Geometrical dimensions of mobile disc, (a) outside diameter, D_B , (b) sectional view of the coning angle, β .

section than position 2'. Consequently, the cutting force is greater in position 1' than in position 2', therefore, a higher energy consumption is expected. Therefore, it is concluded that the position 1' is the most critical and therefore needs to be considered. So, the sphere describes the curve a , see Fig. 8.

For the analysis of movement and the resulting force of the sphere during the disintegration process, some authors assume that the path of the particle follows a logarithmic spiral [2].

The curve a , in Fig. 8, occupy new radial positions which are determined by Eq. (6). It is deduced from Eq. (4).

$$\bar{r}(\theta) = \frac{D_B}{2} - (R(\theta) + e + e_s) \cot \alpha, \quad (6)$$

where $\bar{r}(\theta)$ is the radial position of the sphere during the disintegration process, $R(\theta)$ is the radius of the sphere that varies in each cut.

Determining of the physical and mechanical properties of the product is essential for producing suitable design of a machine [10] and to know the consumed power. In this paper, the product is sweet corn, its shear strength is 300 kN/m², and its density is 600 kg/m³ (It was determined experimentally).

To calculate the forces on the discs is necessary to define some geometry magnitudes; D_B is 160 mm, $R_{(0)}$ is 9 mm, e is 1 mm and e_s is 1 mm $R_{(0)}$ is the radius of the sweet corn that enter to the disc. The diameter is 18 mm, it was calculated as the geometric average of 40 sweet corns, and it was evaluated according to studies realized by Müller [13].

To find $R(\theta)$, it is developed a graph of the sphere radius vs time, according to the conservative particle possible trajectory (curve a). From the curve a , an "r" function is obtained, which interestingly fits an exponential, and if the graph is represented in a polar graph $r(\theta)$ vs θ , it results a logarithmic spiral, see Fig. 9.

Deriving the "r" function, the relative acceleration and relative velocity of the sphere respect to the disc are obtained, and if they are replaced in Eq. (3), the total acceleration of the

sphere is obtained, and thus also the resultant force \vec{F}_R .

$$\vec{F}_R = M(a_{\text{coriolis}}, a_{B/A} - a_{\text{centripeta}}, 0), \quad (7)$$

where M is the sphere mass.

To find the force \vec{R}_c , it is decomposed in its two components $R_c \sin(\beta_n)$ and $R_c \cos(\beta_n)$, where β_n is the angle between \vec{R}_c and the Z' axis, see Fig. 10. The component $R_c \sin(\beta_n)$ produces the cutting force on the sphere. Eq. (8) shows the relation between the shear strength and the force \vec{R}_c .

$$\tau * A = R_c \sin(\beta_n), \quad (8)$$

From Fig. 10 and Eq. (8) can be seen that R_c can be expressed as follows.

$$\vec{R}_c = \left(\tau A \sin(\alpha_n), \tau A \cos(\alpha_n), -\tau \frac{A}{\tan(\beta_n)} \right), \quad (9)$$

where τ is the shear strength, A is the area of the section where the cutting occurs, α_n is the angle determined by the Y' axis and the direction of the cutting force.

The angles β_n and α_n are determined with the disc geometry, distance of separation between discs and sphere radius according to Eqs. (10) and (11).

$$\beta_n = \cos^{-1} \left(\frac{R-s}{R} \right), \quad (10)$$

$$\alpha_n = \cos^{-1} \left(\frac{R \sin d_n}{D_b - R \cot(\alpha) + (e_s + s) \cot(\alpha)} \right), \quad (11)$$

where R is the sphere radius.

So far, the forces \vec{R}_c y \vec{F}_R can be obtained, and the unknown forces \vec{N}_1, \vec{N}_2 , and \vec{E} need to be defined; these forces can be expressed in terms of its magnitude and unit vector, thanks to the given disc geometry.

$$\vec{N}_1 = (-N_1 \cos \alpha, -N_1 \sin \alpha \cos \alpha, -N_1 \sin^2 \alpha), \quad (12)$$

$$\vec{N}_2 = (0, -N_2 \sin 2\alpha, N_2 (-\cos 2\alpha)). \quad (13)$$

Substituting the values \vec{R}_c y \vec{F}_R in Eq. (1), the unknowns \vec{N}_1, \vec{N}_2 y \vec{E} can be estimated for the last sphere which is about to leave the disc (sphere "n"), see Fig. 11.

For the sphere "n-1", it is noted that $\vec{E}_{N_{n-1}}$ is equal to \vec{E}_n , see Fig. 12.

This time the force $\vec{E}_{N_{n-1}}$ is known and it is replaced in Eq. (1). Now, the unknown forces are $\vec{N}_{1_{n-1}}, \vec{N}_{2_{n-1}}$ y $\vec{E}_{N_{n-1}}$, which can be obtained using Eq. (2), and so on and so forth.

It is necessary to know the number of spheres per disc channel, it can be obtained according to Eq. (14).

$$N_{\text{spheres}} \leq \frac{\log \left(\left(\frac{(1-a)(R_1 - e_s - s)}{\sin(\alpha)} - 2 * R_1 + R_1 * (1-a) \right) / (-2 * R_1 - R_1 * \frac{1-a}{a}) \right)}{\log a}, \quad (14)$$

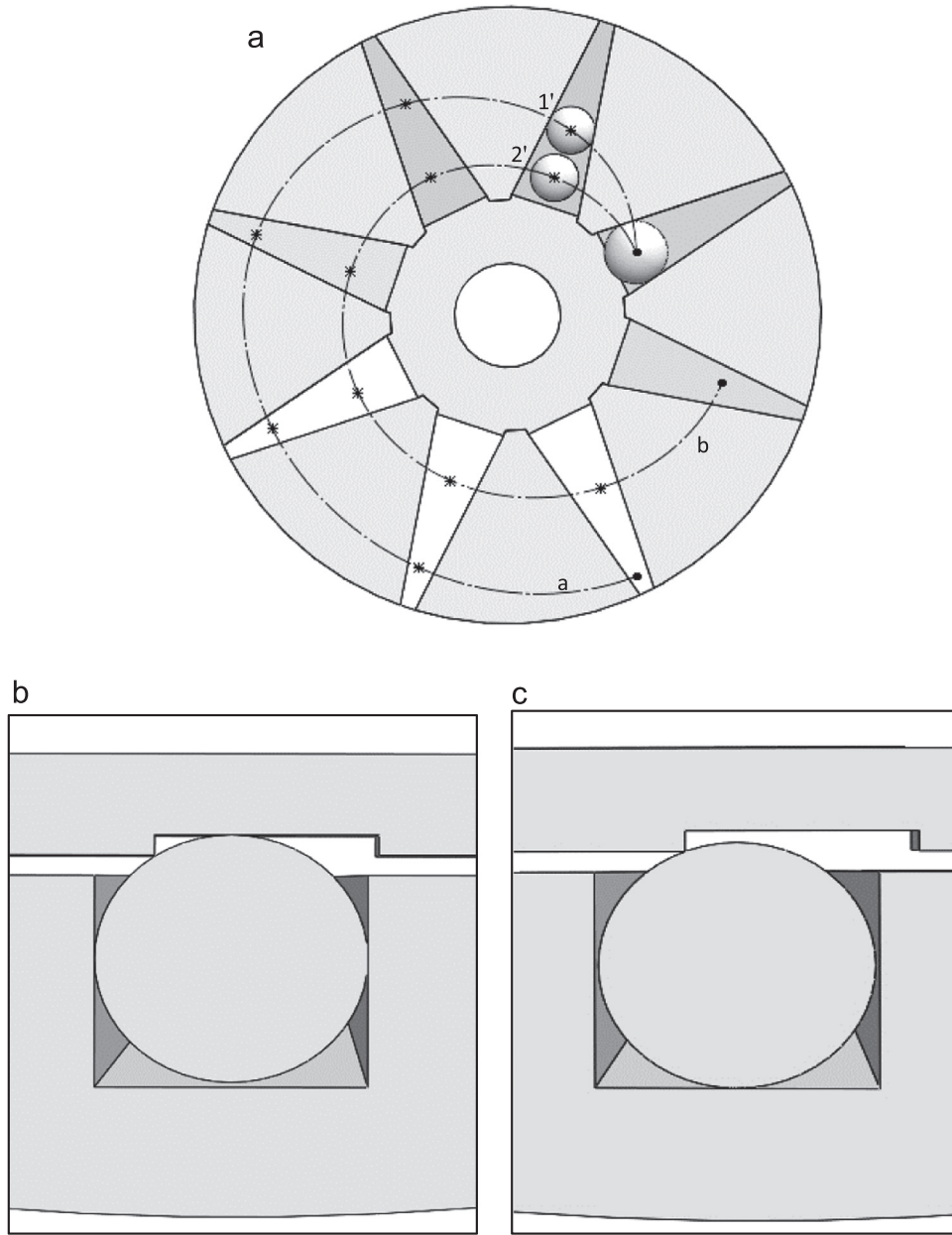


Fig. 8. Trajectory of the particle and possible scenarios of cutting, (a) Particle possible trajectories in the process of disintegration, (b) sectional view, position 1', (c) sectional view, position 2'.

$$a = \frac{1 - \sin(\alpha)}{1 + \sin(\alpha)}, \tag{15}$$

where R_1 is the radius of the sphere with which enters to the disc.

The total power is calculated by the sum of the power produced by each force \vec{N}_1 on the mobile disc, see Eq. (16).

$$Pot_{Total} = N_{Channels} \cdot \sum_{i=1}^{N_{Spheres}} \left(\omega \cdot (\mathbf{r} \times -\vec{N}_1)_i \right) \tag{16}$$

where $N_{Channels}$ is the number of channels on the disc, $N_{Spheres}$ is the number of spheres per disc channel, ω is the mobile disc angular velocity and \mathbf{r} is the distance from the contact point in which operates \vec{N}_1 to z axis.

3. Results and discussions

The presented methodology can be evaluated at several steps; even sensor can be utilized in order to better improve the estimation of fundamental parameters which are of paramount importance to estimate the required power in disc crusher during the milling process. Consequently, further research work in this topic can be performed to improve even more the present methodology. Interesting papers on food plants [14,15] could be used as guide to accomplish this task.

From the formulations described above it can be verified that the forces can be expressed as functions that vary over time. The total power consumed can be expressed as a function of such forces. To evaluate the functionality of the discs, a

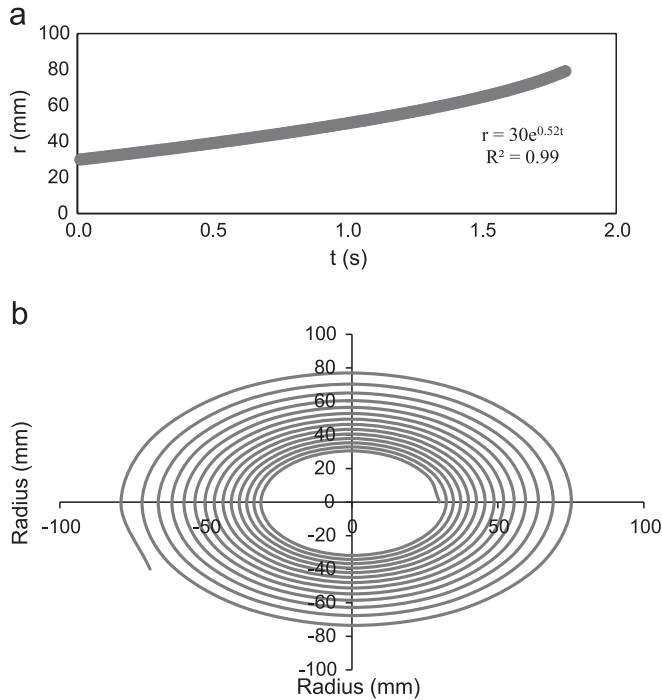


Fig. 9. (a) Radial position vs time, (b) logarithmic spiral describing the motion of the particle during the disintegration process.

module test of disc crushers was constructed. The discs were obtained using a 3D printing machine, see Fig. 13.

Good results in terms of particle size and viscosity for sweet corn were obtained. Results from the present methodology suggest that to start the disintegration process, a thrust force is needed which is important for the design of disc crusher machines, see Fig. 14.

In order to evaluate the final output of the present methodology, which is the required power in disc crushers, Eq. (16) is used to estimate the required disc crush power during the milling process of sweet corn. In order to fulfill the requirements to estimate such power, experimental procedures to characterize sweet corn were performed. The shear stress results was around $300,000 \text{ kg/m}^2$ and the density around 600 kg/m^3 .

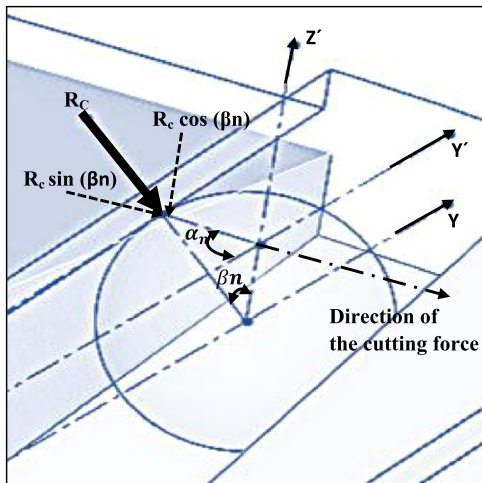


Fig. 10. Components of the force \vec{R}_c .

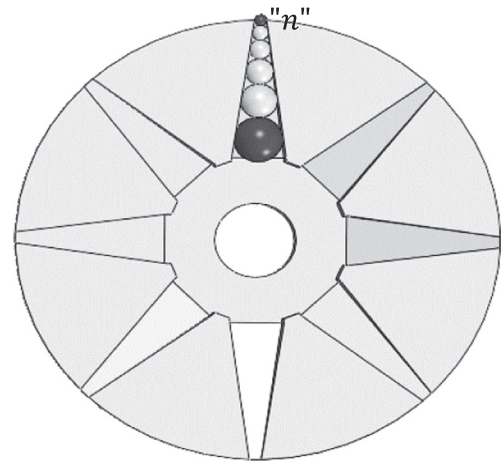


Fig. 11. Sphere about to leave the disc.

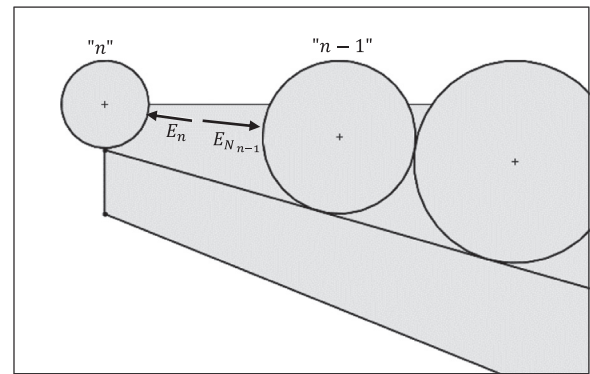


Fig. 12. Action and reaction forces between the sphere "n" and the sphere n-1.

Overall, the estimated required power in the disc crushers varies from 0.74 to 0.77 HP, and depending on the power transmission system, the full required power in the engine is higher. Moreover, the engine power of existing mill machines in the industry of sweet corn milling at low production level is between 0.5 and 1 HP which is very close to the obtained results in this paper.



Fig. 13. Test Module disc crushers.



Fig. 14. Experimental test of square section disc.

In this paper the disc channel section is square, but it is possible to analyze other kind of sections and other kind of channel, in this study the channels are radial, but in future research studies the channel could be logarithmic. Moreover, it is worth to mention that frictional forces were not considered in this paper, this force is important because it allow to estimate the heat produced in the milling process. The heat produced in discs is very important since the properties of food vary due to high temperature during the milling process.

Theoretical analysis of the estimation of required power in disc crushers in a milling process need to be further verified experimentally at several steps during the milling process since there are a lot parameters to be evaluated such as sweet corn temperature, RPM, forces on the disc, monitoring of the distance between the flat faces of the discs to avoid seed corn pollution, etc.

Since this paper present a computational methodology to calculate the required power in disc crushers, the full performance of the machine being built in the context of the present project, and further information of the experimental test on the designed machine may be discusses in future research work.

4. Conclusions

This paper presents a methodology for the estimation of power consumption in a disc crusher disintegration process. The study covers the dynamic analysis of crush forces acting on the particles and the mobile disc. The forces can be expressed as functions that vary over time. The total power consumed can be expressed as a function of such crush forces. This methodology allows verifying that a thrust force is needed to start the disintegration process, this aspect is important when designing disc crusher machines.

The obtained results have provided a preliminary confirmation of the capability of the present methodology to estimate the

required power in disc crushers since the theoretical results are in the range of industrial data, and some theoretical assumptions as the thrust force was experimentally verified. It is important to remark that further research works in this topic needs to be carried out.

Acknowledgments

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