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Estimation of the power consumed in disc crushers using a static analysis

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Abstract

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The classic forms of disc crusher disintegration simulations are faced using methods that associate their analysis to the stochastic nature of the process. This paper introduces a method that can solve the disc crusher disintegration simulation with a partially deterministic manner. The paper presents the result of the thrust force and the consumed power of disc crushers which is estimated by a static analysis of forces that interact within the discs (fixed and mobile). This methodology uses the calculation of forces and directions involved in the process of particle fragmentation. The fragmented material is characterized. The fragmentation process is simulated with the help of the computer programme to finally estimate the consumed power. For comparison purposes, two types of disc crushers are designed to determine the capability of the method after assessing the results.

Crushing is a key process in the industry; there are rollers, rods and others crushers. They are used in the agricultural, wood, mining and chemical industries (Oduntan, Omitoyin 2015). There are other fields where this methodology could be also applied, for example: milling of oil (Leone 2014), the effects of different mechanical crushers in the process of olive paste (Leone et al. 2015), the studies on the influence of physical properties of seeds on shelling performance using a disc mill (Romuli et al. 2015), the design and redesign of machines that work with olive paste (Tamborrino et al. 2014; Leone et al. 2016), the improvement of sugar yields from corn stover using sequential hot water pre-treatment and disc milling (Kim et al. 2016). These are recent and important studies that suggest the need to fully understand the physics and mechanics in the design and performance of new types of disc crushers. Many discs have been designed under laws of traditional crushing. For example, energy laws describe the relationship between the necessary energy and obtained size reduction, which expresses that the required energy for a given disintegration process is exponentially proportional to the size of the particle. Similarly, the Kick law states that the absorbed energy to

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produce similar changes in the configuration of geometrically similar bodies varies with the body's volume or mass. The Rittinger law states that the necessary work in a disintegration process is proportional to the increase of the surface produced. A third theory is called the Bond law and states that the work required in the process of disintegration is proportional to the square root of the diameter of the produced particles (Alfondo 2003). All these laws are aimed at crushing the raw material. However, for particles in which disintegration process is not efficient by a break process, it is preferable to work mainly by shear.

Disintegration process is considered as a stochastic process and simulation of the disintegration process goes through stochastic models, which are linked to the statistical theory such as the "Monte Carlo Method", mathematical methods as the Discrete Element Method (DEM), Finite Element Method (FEM) or using the "Markov Chains" among others. Mathematical models of the kinetics of decay are classified according to the class of the Markov process (Zueva et al. 2010). To solve these problems of simulating processes, it is necessary to create a bank of mathematical models of the processes of technology that can be solved using a computer calculation (Feller 1968). It is not known to what extent it is possible to develop a method that can simulate this process associating it to a partially deterministic character; in other words, to determine the factors present in a disintegration process and conditions in a process that give the expected results. Prior to the development of this methodology, the calculation of the required power for the disintegration process within the disc crushers is performed following the same logic as analysis of Aguedo (1991). This study was conducted to determine the static forces involved within the disc crushers. However, the calculus did not contemplate neither dynamic forces nor the support of computer modelling.

This paper introduces a methodology to estimate the thrust force and the consumed power of disc crushers by a static analysis of the forces that interact within the discs (fixed and mobile). It introduces the factors involved in the disintegration process within disc crushers in order to design a disc model with particular geometry. Finally, the methodology can determine the required power using mathematical calculation which can be easily programed.

MATERIAL AND METHODS

A conventional disc crusher machine has a feed hopper, conduit toward discs, power transmission system and discs. The disc crusher that was used for this study has two concentric discs, one fixed and one rotating; and they are at an adjustable distance. The disintegration process in disc crushers begins when the raw materials, in this case sweet corns, are poured into the inlet of the machine, which is usually a hopper. Then the sweet corns enter a chamber where they are driven to the surface of the disc by a screw. The crushed corns enter on the discs and disintegration occurs. The above description corresponds to the disintegration process of the particles in the disc crushers used for this analysis. Fig. 1 shows the scheme of the disc crusher machine.

The driving force or power needed for crushing certain amount of grain depends on many factors; it depends on the crushing system, layout of machinery components, the type of construction system, and finally the engine system. The shortage of lubrication, poor maintenance of machinery, transmissions, lack of cleanliness, etc., all these factors cause higher power consumption (Richardson 1950).

The study of disintegration process started assuming that it is possible to simulate the process following a partially deterministic manner. This is

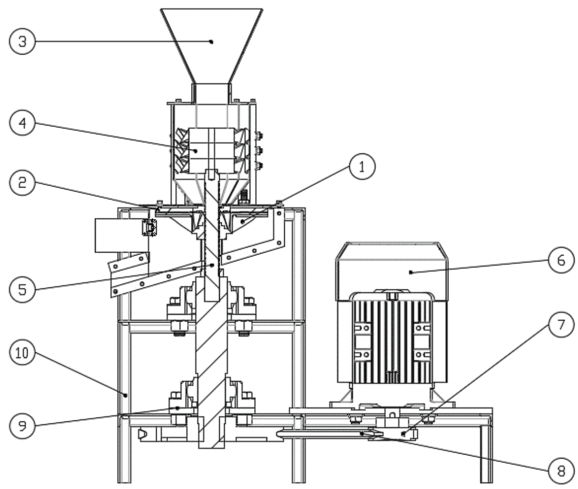

Fig. 1. Disc crusher machine
1 – mobile disc, 2 – fixed disc, 3 – feed hopper, 4 – cutting chamber, 5 – shaft, 6 – electric engine, 7 – transmission pulley, 8 – transmission belt, 9 – bearing, 10 – structure support

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(a) (b)

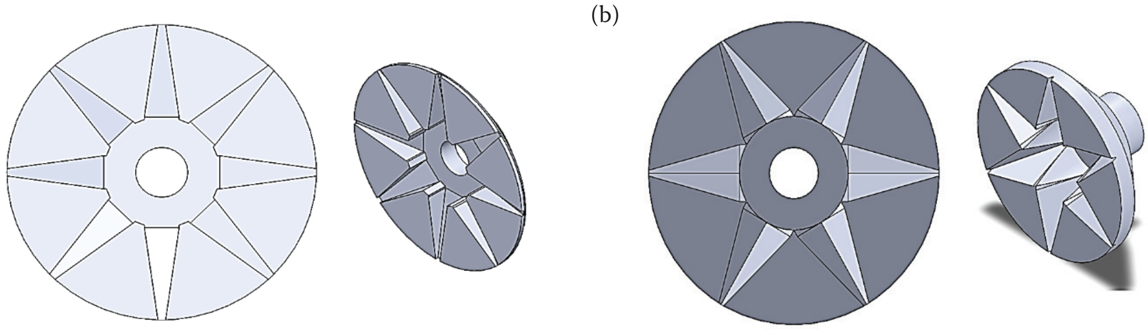

Fig. 2. Disc with (a) eight radial channels of square section, and (b) six radial channels of triangular section

possible if the intrinsic characteristics of the product are known and the discs are conditioned to a geometry that fits properly. A particle inside the discs has speed and acceleration. These relationships give rise to a more complex analysis that includes fictitious forces such as centripetal or Coriolis forces; these forces are not considered since it is a static calculation. The trajectory that describes a particle can be a logarithmic spiral (Vaculík et al. 2013). This conclusion is taken into account for the design of the disc geometry. For a proper arrangement of variables involved in the disintegration process, the raw material is characterized as spherical bodies with uniform physical properties that do not depend on its mass.

The analysis begins proposing suitable geometry. The discs are designed considering the movement of the raw material through the channels. With the movement of the mobile disc, the spheres are accommodated in the disc channels. As a result, a section of sphere volume protrudes on the mobile disc surface and the fixed disc cuts it. For this purpose, two geometries are studied. The dimensions of the discs depend on the initial volume and the final volume of the spherical body. The centres of these spherical bodies are contained in a line. This line forms a determined angle with the plane of the disc surface. The number of channels of the disc depends on the volume of the first spherical body. Two types of geometries for the mobile disc are designed, one square section and the other triangular. In both types, the section is reduced as it approaches to the external limit of the disc (Fig. 2).

The spherical bodies are tangent to the channel walls and between them. The accommodation of the spherical bodies within the channel enables to situate, according to their volume, each spherical body in a determinate position. Naturally, the sphere of larger volume is located at the entrance of the channel, i.e. closest to the centre of the disc. The sphere of higher volume is obviously located at the first position and the smaller at the last position, see Fig. 3.

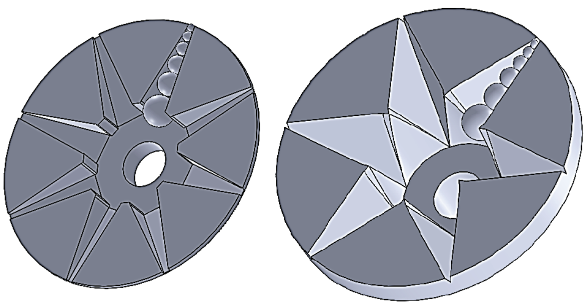

Fig. 3. Accommodation of the spherical bodies on the surface of the channel of square section (left) and triangular section (right)

The next step is to calculate the number of spherical bodies that can enter the channel, Eq. (1) (Aguedo 1991). This amount is verifiable through computational modelling of the disc. The 3D modelling of the discs in a CAD program allows observing and determining the forces and distributions of the spheres, which are not found in Aguedo's works (1991):

$$n \leq \frac{\ln\left(\dfrac{\left(\frac{2\sin\alpha}{1+\sin\alpha}\right)\frac{(R_1-R_n)}{\sin\alpha} - 2R_1 + R_1\left(\frac{2\sin\alpha}{1+\sin\alpha}\right)}{-2R_1 - R_1\frac{2\sin\alpha}{1-\sin\alpha}}\right)}{\ln\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right)} \tag{1}$$

$$\alpha = a\tan\frac{R_1-R_n}{D_M-D_m} \tag{2}$$

where: $\boldsymbol{n}$ – number of spheres that may be within the channel; $\boldsymbol{R_1}$ and $\boldsymbol{R_n}$ – radii of the first and last spherical bodies, respectively; $\boldsymbol{\alpha}$ – angle between the line that joins the centres of the spheres and a horizontal plane (Fig. 4), the angle $\boldsymbol{\alpha}$ is independent of the type of chan-

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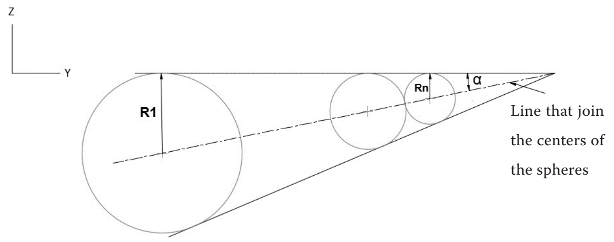

Fig. 4. Angle α
***R*1** – radius of the smaller sphere

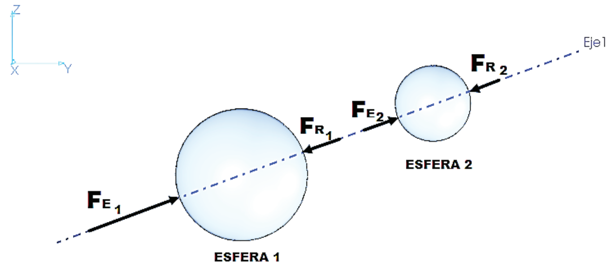

Fig. 6. Representation of the forces $\boldsymbol{F_E}$ and $\boldsymbol{F_R}$ for each sphere
$\boldsymbol{F_E}$ – thrust force; $\boldsymbol{F_R}$ – reaction force resulting from the interaction between spherical bodies

nel section; $\boldsymbol{D_M}$, $\boldsymbol{D_m}$ – outer and inner diameters of the mobile disc respectively (Fig. 5)

To establish the direction and magnitude of the forces acting on each spherical body, it is necessary to define two reference systems. Origin of the first reference system coincides with the centre of the mobile disc surface. The second reference system *xyz* is also located on the surface of the mobile disc, but it rotates with the same angular velocity as the mobile disc. The second reference system is used to study the position of the spheres through the disc channels.

The thrust force $\boldsymbol{F_E}$, which is directed to the centre of each particle, is located on a plane parallel to the *YZ* plane. The purpose of this force is to push the spherical bodies through each channel (Fig. 6). The orientation of this force is independent of the type of disc section. $\boldsymbol{F_R}$ is the reaction force resulting from the interaction between spherical bodies. The forces $\boldsymbol{F_E}$ and $\boldsymbol{F_R}$ have opposite directions, but the same magnitude. $\boldsymbol{F_R}$ pushes the body toward the centre of the disc (Fig. 6). The forces $\boldsymbol{F_R}$ and $\boldsymbol{F_E}$ are not considered in the work performed by Aguedo (1991).

The forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are defined as the reaction forces on the spheres due to contact with the wall of the channels (Fig. 7), Eq. (3) and Eq. (4) represent the forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in vectorial form. In addition, modules of these forces are different for each spherical body.

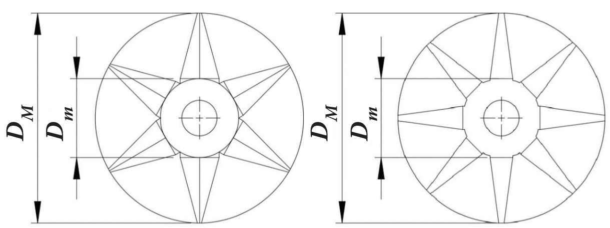

Fig.5. Geometry discs of triangular section (a) and square section (b)
DM*, *Dm – outer and inner diameters of the mobile disc respectively

$$\vec{F}_{1i} = (-F_{1i}\,\mu_{1x}, -F_{1i}\,\mu_{1y}, -F_{1i}\,\mu_{1z}) \tag{3}$$

$$\vec{F}_{2i} = (0\,, -F_{2i}\,\mu_{2y}, F_{2i}\,\mu_{2z}) \tag{4}$$

where: $\boldsymbol{\mu_1}$, $\boldsymbol{\mu_2}$ – direction cosines for the forces $\vec{F}_{1i}$ and $\vec{F}_{2i}$, respectively; $\boldsymbol{F_{1i}}$, $\boldsymbol{F_{2i}}$ – modules of $\vec{F}_{1i}$ and $\vec{F}_{2i}$

It is worth mentioning that the direction cosines depend on α; details are shown in Appendix A.1.

The force $\boldsymbol{F_c}$ is due to the interaction between the spherical body and the fixed disc. This force is responsible for the sphere volume reduction. In addition, it is responsible for the position of the bodies through the disc channel. Finally, $\boldsymbol{F_c}$ depends on the radius, density and shear strength of the spherical body (Fig. 8a).

$$\vec{F}_{Ci} = (\tau A_i \sin(e_i), \tau A_i \cos(e_i), -\tau A_i \,\mathrm{ctg}(d_i)) \tag{5}$$

where: $\vec{F}_{Ci}$ – force vector defined for each spherical body; $\boldsymbol{\tau}$ – shear strength of spherical bodies; $\boldsymbol{A_i}$ – area of the section where the cutting occurs; $\boldsymbol{e_i}$ – angle between the direction of the force τA_i and an axis parallel to *y* axis; $\boldsymbol{d_i}$ – angle between $\boldsymbol{F_C}$ and an axis parallel to $\boldsymbol{z}$ axis (Fig. 7b)

It is worth mentioning that $\boldsymbol{e_i}$ and $\boldsymbol{d_i}$ are different for each spherical body. This relationship is valid for both channel sections. *C* represents the circle that encloses the area *A*. Fig. 8b shows the components of the force $\boldsymbol{F_C}$. It is important to mention that the component $\boldsymbol{F_{Cx}}$ is not equal to τA. F_{gi} is the gravity force, which only depends of the volume and density of each spherical body.

So far, the forces are obtained in terms of variables that depend on the channel geometry. A simple computer program, which is based in Eq. (6), is coded to obtain numerical results.

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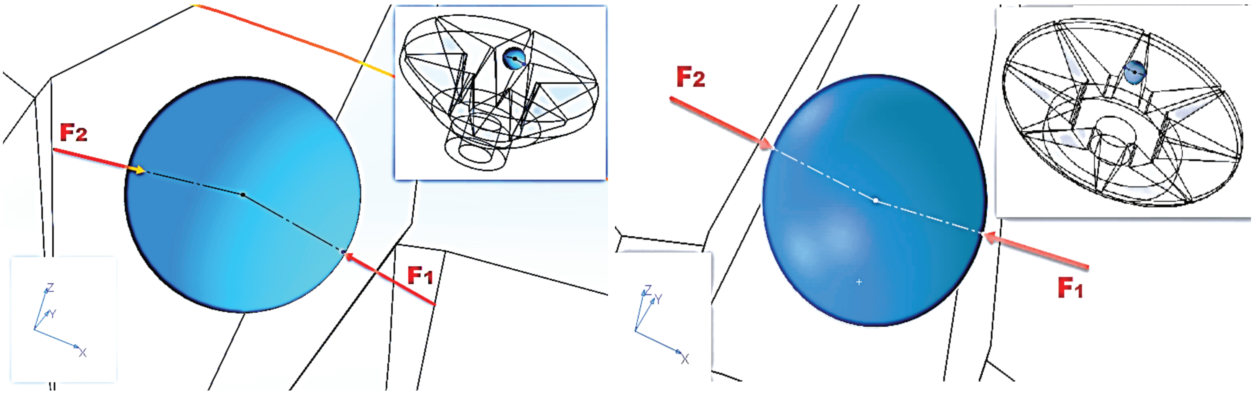

Fig. 7. Representation of the forces F_1 and F_2 for a disc radial channel of (a) triangular and (b) square section
F_1, F_2 – reaction forces on the spheres due to contact with the wall of the channels

$$\vec{F}_E + \vec{F}_R + \vec{F}_1 + \vec{F}_2 + \vec{F}_C + \vec{F}_g = 0 \tag{6}$$

From Eq. (6), it is possible to obtain three equations in *x*, *y* and *z* directions. Each equation is located in a matrix form according to Eq. (7).

$$\boldsymbol{A} \times \boldsymbol{X} = \boldsymbol{B} \tag{7}$$

$$\boldsymbol{A} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \qquad \boldsymbol{X} = \begin{bmatrix} F_1 \\ F_2 \\ F_E \end{bmatrix} \ \boldsymbol{B} = \begin{bmatrix} -F_{Cx} \\ -F_{Cy} - F_R \\ Fg - F_{Cz} \end{bmatrix}$$

where: the values of the matrix $\boldsymbol{A}$ are constant for all the spherical bodies and are defined according to the disc geometry, these values are defined in Appendix A.2. The values of the matrix $\boldsymbol{X}$ correspond to the modules of $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_E$. $\boldsymbol{B}$ is a matrix of the known terms. $\vec{F}_C$ and $\vec{F}_g$ can be obtained directly. For the last sphere, the force $\vec{F}_R$ is zero, and because of this, it is possible to solve Eq. (7). Therefore, the forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_E$ can be calculated. So the force $\vec{F}_R$ for the penultimate sphere is known. Following the proposed logic and with the help of computational calculation, it is possible to solve Eq. (7) for each sphere.

Once forces acting on each sphere are known, it is possible to calculate the power required to move the mobile disc trough the drive shaft. Eq. (8) shows the power consumed in each sphere:

$$\text{Pot} = \vec{w} \times (\vec{r}_i \times \vec{F}_i) \tag{8}$$

where: $\vec{w}$ – angular velocity vector with which the mobile disc rotates; $\vec{F}$ – applied force on the surface of the mobile disc; $\vec{r}$ – the vector from the centre of the mobile disc toward the point of application of force $\vec{F}$.

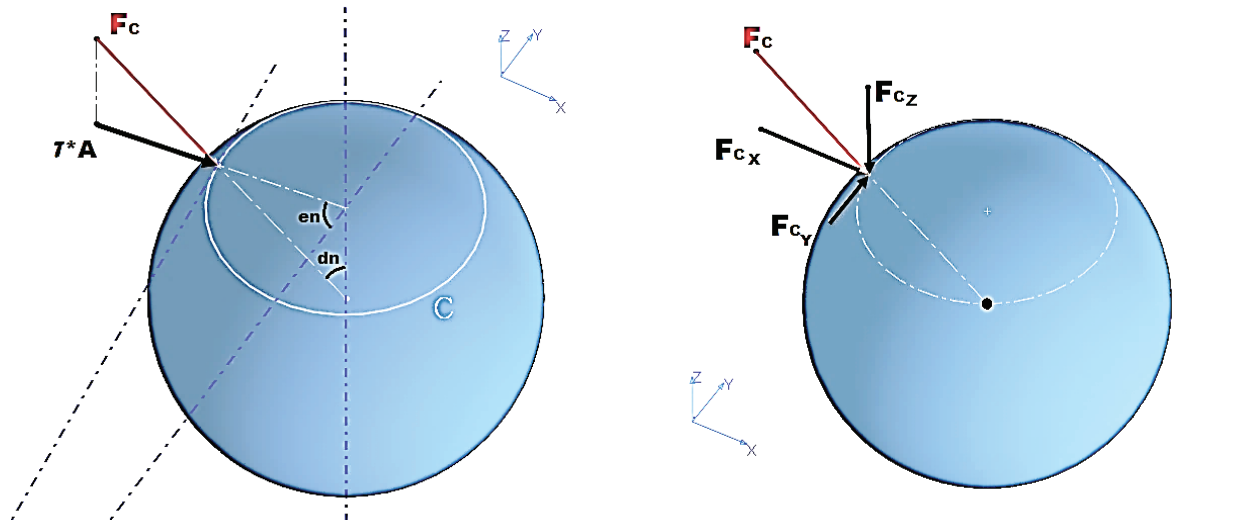

Fig. 8. Factors that determine the force F_C (a) and decomposition of force F in the system *xyz* (b)

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Table 1. Forces for each sphere positioned within the disc channel

	Square section			Triangular section		
i	F_E (N)	F_1 (N)	F_2 (N)	F_E (N)	F_1 (N)	F_2 (N)
1	24.73	16.02	32.67	11.32	42.26	23.75
2	15.84	11.97	20.47	7.53	27.55	13.73
3	9.55	8.84	12.58	4.72	17.76	7.56
4	5.37	6.45	7.53	2.84	11.28	3.82
5	2.68	4.65	4.32	2.49	6.99	1.62
6	1.005	3.29	2.31	0.86	4.19	0.39

i – accountant; F_E – thrust force; F_1, F_2 – forces that produce torque

The forces $\vec{F_1}$ and $\vec{F_2}$ produce torque. Eq. (8) is applied on each sphere. Therefore, the total power to be transmitted is the sum of the power calculated in each spherical body, see Eq. (9).

$$\mathrm{Pot}_{\mathrm{TOTAL}} = \Sigma_i^n \vec{w} \times (\vec{r_i} \times -\vec{F_i}) \quad (9)$$

The physical and mechanical properties of agricultural products are essential for producing suitable designs (Müller et al. 2014). For this paper, the product is sweet corn, its shear strength is 300 kN/m^2, and its density is 600 kg/m^3 (those values were determined experimentally).

RESULTS AND DISCUSSION

To obtain the forces acting on the sphere, it is necessary to know some geometric parameters of the discs, such as D_M (160 mm), D_m (60 mm), the radius R_1 of the first body (9 mm), radius for the last sphere R_n (2 mm) and angular velocity w (360 RPM). Using the parameters given in Eq. (1) and Eq. (2), the values of α and n are 8° and 6, respectively, this can be verified graphically using the static computational modelling. Applying Eq. (7), the values of F_E, F_1 and F_2 are obtained (Table 1).

Replacing the forces obtained in Eq. (9), the power consumed in each kind of mobile disc is obtained. In the square section disc, the power is 0.77 HP and in triangular section disc, the power is 1.03 HP. It was noted that it is more appropriate to work with a square section disc, because it consumes less power than a triangular section disc. Nevertheless, the thrust force F_E in a square section disc is greater than the thrust force in a triangular section disc, as it causes difficulties to push the raw material through the disc channel, which was experimentally verified (Fig. 9).

It is worth mentioning that if it is desired to choose an electric engine, it is necessary to add an extra power; it is due to a loss of power in mechanisms that produce the thrust force F_E, and power transmission system.

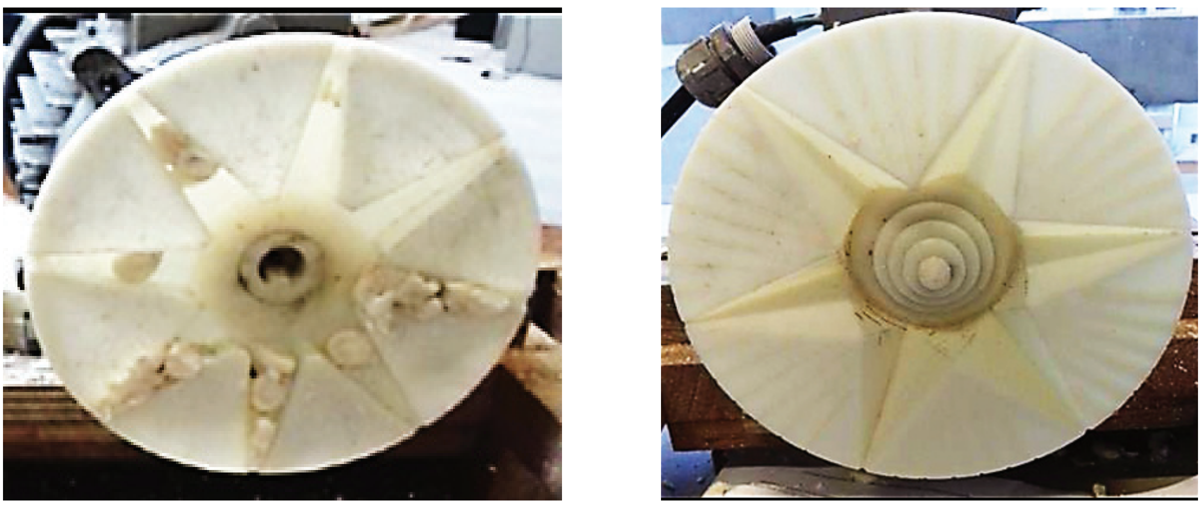

Fig. 9. Experimentally test in discs of square section (left (Zuñiga, Mantari 2017)) and triangular section (right)

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For more reliable results, further studies on dynamic analysis of disc crushers should be performed. However, this static method gives an acceptable estimate of the power consumed in disc crushers, and gives an approximation of the required thrust force to move a raw material through the disc channel. Consequently, further studies need to be carried out in order to improve the present methodology.

CONCLUSION

The proposed method allows calculating the power consumed by disc crushers in a disintegration process. The factors that determine the consumed power are disc geometry and the forces acting on it; it is associated with the geometry of the raw material for milling. This methodology aims at simplifying the way to calculate the power in disc crushers and it gives a close estimation of real industrial data; thus it can be used as a preliminary indicator of the power required in disc crushers.

Appendix

A.1. Values of direction cosines of Eqs (3 and 4)

Values for a disc channel of square section

$\mu_{1x} = \cos(\alpha)$

$\mu_{1y} = \cos(\alpha)\sin(\alpha)$

$\mu_{1z} = 1 - \cos(\alpha)^2$

$\mu_{2x} = 0$

$\mu_{2y} = 2\cos(\alpha)\sin(\alpha)$

$\mu_{2z} = 1 - 2\cos(\alpha)^2$

Values for a disc channel of triangular section

$\mu_{1x} = \mu_{2x} = \sqrt{3}/2\cos(\alpha)$

$\mu_{1y} = \mu_{2y} = 3/2\cos(\alpha)\sin(\alpha)$

$\mu_{1z} = \mu_{2z} = 1 - 3/2\cos(\alpha)^2$

A.2. Values of the elements of matrix A, which belong to Eq. (7)

Values for a disc channel of triangular section

$k_{11} = -\sqrt{3}/2\cos(\alpha)$

$k_{12} = \sqrt{3}/2\cos(\alpha)$

$k_{13} = 0$

$k_{21} = -3/2\cos(\alpha)\sin(\alpha)$

$k_{22} = -3/2\cos(\alpha)\sin(\alpha)$

$k_{23} = 1$

$k_{31} = 1 - 3/2\cos(\alpha)^2$

$k_{32} = 3/2\cos(\alpha) - 1$

$k_{33} = 0$

Values for a disc channel of square section

$k_{11} = -\cos(\alpha)$

$k_{12} = 0$

$k_{13} = 0$

$k_{21} = \cos(\alpha)\sin(\alpha)$

$k_{22} = 2\cos(\alpha)\sin(\alpha)$

$k_{23} = 1$

$k_{31} = \cos(\alpha)^2 - 1$

$k_{32} = 2\cos(\alpha)^2 - 1$

$k_{33} = 0$

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