2D Amplitude-Modulation Frequency-Modulation - based Method for Motion Estimation

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***Abstract*—We present a new approach to compute the motion estimation in digital videos using 2D amplitude-modulation frequency-modulation (AM-FM) methods. The optical flow vectors are computed using an iteratively reweighted norm for total variation (IRN-TV) algorithm. We compare the proposed method using synthetic videos versus a previous three-dimensional AM-FM based method and available motion estimation methods available in free and commercial software. The results are promising, in terms of accuracy, producing a full density estimation with more accurate results than the other methods.**

***Index Terms*—motion estimation, optical flow, amplitude-modulation frequency-modulation**

I. Introduction

The development of perception-based motion estimation methods based on multiscale amplitude-modulation frequency-modulation (AM-FM) methods has a strong interest in digital video processing [1]. The standard use of perception-based methods for motion estimation methods rely on the use of Gabor filters, requiring a large number of frames to be computed [1]. In this paper, we present a new approach that uses a 2D multiscale AM-FM method to estimate optical flow motion using two video frames only. This the next step of the methodology discussed in [2].

Our motivation for the new perception-based approach comes from the need to maintain compatibility with the majority of optical flow motion methods that are based on a small number of video frames [3], [4]. In [5], the authors describe the motion estimation methods applied to biomedical imaging applications. The methods described in [5], for example [6], require smooth variations that have a very difficult time with ultrasound videos that are dominated by speckle noise. For speckled ultrasound videos, there are no perception-based methods that can produce motion estimation estimates at every single pixel [7].

In this paper, we propose a new approach to a motion estimation based on 2D AM-FM that goes beyond the 3D AM-FM methodology described in [8] and the 2D IF-based methods described in [2]. This method only requires two video frames: $I(x, y, t)$ and $I(x, y, t + 1)$. This method takes advantage of both the instantaneous amplitude (IA) and instantaneous frequency (IF) estimates compared to [2]. The problem is solved using a total variation formulation of our approach. We present comparative results on synthetic videos where we show that the proposed approach provides better estimates than many other standard methods. We compare the method versus those available in free and commercial software such as Matlab, OpenCV and CUDA: Horn-Schunck [9], Lucas-Kanade [10], Gunnar Farnebäck [11] and TVL1 [12].

This manuscript is organized as follows. We provide background on related work in section II. In section III, we present the methods proposed. The results and discussion of the proposed method is presented in section IV. Finally, we present the conclusions and future work in section V.

II. Background

Optical flow is the distribution of apparent velocities of movements of brightness patterns in an image [9]. It is the most challenging version of motion estimation (ME) because it is about to compute an independent estimate of motion at each pixel [13], or voxel. Ideally, the optical flow is equal to the motion field. However, this does not always happen as these cases [9]: (i) a motion camera recording a stationary object produces optical flow, or (ii) a sphere of uniform intensity in rotation does not produce optical flow. Thus, optical flow estimation is still one of the problems in computer vision [14]. In digital videos, the motion estimation vectors are not recognized in a total optical flow field. The standard methods have certain parameters that affect the final accuracy. There are common assumptions of an image to use optical flow methods such as grey value constancy, gradient constancy and smoothness assumptions [13].

In the last decades, the original optical flow estimation methods have been changing until give rise to new methods. The basic Horn-Schunck [9] and Lucas-Kanade [[10] methods have been modified to improve the quality of optical flow estimation [14]. Nevertheless, these changes are not sufficient to define accurate motion estimation. In the next lines, we will describe the basic AM-FM definition and other methods to be tested (methods available in Matlab, OpenCV and CUDA).

A. AM-FM representation

A digital video is represented as $I(\mathbf{z}, t)$, where $\mathbf{z} = (x, y)$, in terms of a collection of AM-FM components given by:

$$I(\mathbf{z}, t) = \sum_{n=1}^{n=M} a_n(\mathbf{z}, t) \cos(\varphi_n(\mathbf{z}, t)), \tag{1}$$

where $n = 1, 2, \ldots, M$ denotes the different AM-FM components, $a_n(\mathbf{z}, t)\cos(\varphi_n(\mathbf{z}, t))$ denotes the n-th AM-FM component, a_n denotes the n-th instantaneous amplitude (IA) component, and φ_n denotes the n-th instantaneous phase (IP) component. The instantaneous frequency (IF) is defined in terms of the gradient of the instantaneous phase: $\nabla\varphi_n$. Some examples of the use of AM-FM models in medical imaging are in [15].

For a 3D AM-FM motion estimation method [6], we consider a single AM-FM component given by $I(\mathbf{z}, t) = a(\mathbf{z}, t)\exp(j\varphi(\mathbf{z}, t))$. Next, we consider the Optical Flow Constraint Equation (OFCE, [9]) given by

$$\langle \nabla I,\, \boldsymbol{\zeta} \rangle + \frac{\partial}{\partial t} I = 0, \tag{2}$$

where $\boldsymbol{\zeta} = (u, v)$, $\langle . \rangle$ represents inner product, and we apply it to $I(\mathbf{z}, t)$. Then, the AM constraint is described by analyzing the real part $\langle \nabla a,\, \boldsymbol{\zeta} \rangle + \frac{\partial}{\partial t} a = 0$, and similarly, the FM constraint by analyzing the imaginary part $\langle \nabla \varphi,\, \boldsymbol{\zeta} \rangle + \frac{\partial}{\partial t}\varphi = 0$.

One of the advantages here is the use of two equations per voxel. The motion estimation vectors $u(x, y)$ and $v(x, y)$ are computed using an iterative method based on finite difference approximations [1].

B. Horn-Schunck method

The Horn-Schunck (HS) method assumes that image brightness constraint is constant and the frames have a global smoothness. The HS algorithm has two processes to follow in order to find the desired results. The first one is to estimate the partial derivatives of the frames. The second is to minimize the error of the sum of the partial derivatives because in practice the measurement of brightness can be different at each pixel [9]. The method is based on (2) and it seeks to minimize both $I(x, y, t) = I(x+u, y+v, t+1)$ and $\epsilon = uI_x + vI_v + I_t = 0$ at the same time.

C. Lucas-Kanade method

This method divides the original image in smaller sections and assumes that constant speed exists in every region between frame k and frame $k+1$. Thus, it should satisfy the following equation [10]: $I_{x(Q_i)}u + I_{y(Q_i)}v = -I_{t(Q_i)}$, where Q_i are the pixels inside the section, $I_{x(Q_i)}, I_{y(Q_i)}, I_{t(Q_i)}$ are the partial derivatives of the image I with respect to position x, y and time t, evaluated at the point Q_i and at the current time.

Then, it performs a weighted least-square fit of optical flow constraint equation to a constant model for (u, v) in each section Ω, by minimizing the following equation: $\sum_{x=\Omega} W^2[uI_x + vI_v + I_t]^2$, where W is a window function that emphasizes the constraints at the center of each section.

D. TVL1 method

This method assumes that brightness is constant. So its partial derivatives are equal to 0 [12]: $\nabla I \cdot (u, v) + \partial I/\partial t = 0$. Applying the chain rule and replacing $u(x, y) = (u_1(x, y), u_2(x, y))$, we find the optical flow constraint equation.

E. Gunnar Farnebäck method

This method assumes that the displacement field is only slowly varying. The first equation of this method is in the case that a polynomial undergoes an ideal translation [11]: $f_1(x) = x^T A_1 x + b_1^T x + c_1$. Then construct a new signal f_2 by a global displacement by d: $f_2(x) = f_1(x-d) = (x-d)^T A_1 (x-d) + b_1^T(x-d) + c_1$. By comparison of the coefficients in the quadratic polynomials yields to: $A_2 = A_1$, $b_2 = b_1 - 2A_1 d$, and $c_2 = d^T A_1 d - b_1^T d + c_1$.

III. Methods

A. Single AM-FM Component

We propose to solve the motion estimation problem using only two consecutive frames from a digital video $I(\mathbf{z}, t)$. Let's define $I_1(\mathbf{z}) = I(\mathbf{z}, t)$ as the image intensity function at time t. Also, let's define $I_2(\mathbf{z}) = I(\mathbf{z}, t+1)$ as the image intensity at time $t+1$. Thus, we define the optical flow motion estimation problem for each pixel $\mathbf{z} = (x, y)$ as one of determining the optical flow functions $\boldsymbol{\zeta}(\mathbf{z}) = (u(\mathbf{z}), v(\mathbf{z})) = (u(x, y), v(x, y))$ from I_1 and I_2.

We assume small changes in image intensity as given by $I(\mathbf{z}, t)$ to $I(\mathbf{z}, t+1)$. Next, let's consider the basic model for a single AM-FM component given by:

$$I(\mathbf{z}, t) = a(\mathbf{z}, t)\exp[j\varphi(\mathbf{z}, t)]. \tag{3}$$

Thus, for two consecutive frames we have:

$$I(\mathbf{z}, t+1) = I(\mathbf{z}+\boldsymbol{\zeta}, t) = a(\mathbf{z}+\boldsymbol{\zeta}, t)\exp[j\varphi(\mathbf{z}+\boldsymbol{\zeta}, t)]. \tag{4}$$

Given that $I_k(\mathbf{z}) = a_k(\mathbf{z})\cos\varphi_k(\mathbf{z})$, with $k \in \{1, 2\}$, and based on (2), we have that:

$$\begin{aligned} I_2(\mathbf{z}) &= a_2(\mathbf{z})\cos\varphi_2(\mathbf{z}) \\ &= I_1(\mathbf{z}+\boldsymbol{\zeta}) = a_1(\mathbf{z}+\boldsymbol{\zeta})\cos\varphi_1(\mathbf{z}+\boldsymbol{\zeta}). \end{aligned}$$

In [2], the derivation was based on the FM component by dividing the image output by the IA as given by: $\bar{I}(\mathbf{z}, t) = (a(\mathbf{z}, t)\exp[j\varphi_1(\mathbf{z}, t)])/a(\mathbf{z}, t) = \exp[j\varphi_1(\mathbf{z}, t)]$. The idea is to consider the simplest case given by $\varphi(\mathbf{z}) = \langle \boldsymbol{\omega},\, \mathbf{z} \rangle + c$, where c is a phase constant, $\boldsymbol{\omega} = (\omega_1(\mathbf{z}), \omega_2(\mathbf{z})) = (\omega_1(x, y), \omega_2(x, y))$, $\langle . \rangle$ represents inner product, and $\varphi_1(\mathbf{z}) = \varphi(\mathbf{z})$.

For the proposed method, we consider both the IA and the IF estimates at the same time. For this, we consider the ratio of the two consecutive frames:

$$\begin{aligned} \frac{I(\mathbf{z}, t+1)}{I(\mathbf{z}, t)} &= \frac{a(\mathbf{z}+\boldsymbol{\zeta}, t)}{a(\mathbf{z}, t)}\exp[j(\varphi(\mathbf{z}+\boldsymbol{\zeta}, t) - \varphi(\mathbf{z}, t)] \\ &= \frac{a(\mathbf{z}+\boldsymbol{\zeta}, t)}{a(\mathbf{z}, t)}\exp[j\langle \boldsymbol{\omega},\, \boldsymbol{\zeta} \rangle] \end{aligned}$$

Thus, for an arbitrary instantaneous amplitude, we have the basic equation:

$$\langle \boldsymbol{\omega},\, \boldsymbol{\zeta} \rangle = \mathtt{Arg}\left(\frac{I(\mathbf{z}, t+1)}{I(\mathbf{z}, t)}\right). \tag{5}$$

More generally, by applying (5) to (2) we have the basic linear-phase model approximation given by:

$$\gamma(\mathbf{z}) = \langle \boldsymbol{\phi},\, \boldsymbol{\zeta} \rangle = \texttt{Arg}\left(\frac{I(\mathbf{z}, t+1)}{I(\mathbf{z}, t)} \right), \tag{6}$$

where $\boldsymbol{\phi} = \nabla\varphi(\mathbf{z}) = (\varphi_x(\mathbf{z}), \varphi_y(\mathbf{z}))$ are the x and y components of the IF vectors. Over the entire image, we form the error in solving for (6) using an energy function given by

$$E_{FM} = \sum_{\mathbf{z}} \left[\langle \boldsymbol{\phi},\, \boldsymbol{\zeta} \rangle - \gamma(\mathbf{z}) \right]^2. \tag{7}$$

Since the amplitude is slowly-varying, we can use a Taylor series expansion and assume constancy to get the following approximation:

$$\chi(\mathbf{z}) = \langle \boldsymbol{a},\, \boldsymbol{\zeta} \rangle = a(\mathbf{z}, t+1) - a(\mathbf{z}, t), \tag{8}$$

where $\boldsymbol{a} = (a_x, a_y) = (\partial a/\partial x, \partial a/\partial y)$. Thus, over the entire image, we form the error in solving for (8) using an energy function given by

$$E_{AM} = \sum_{\mathbf{z}} \left[\langle \boldsymbol{a},\, \boldsymbol{\zeta} \rangle - \chi(\mathbf{z}) \right]^2. \tag{9}$$

In general, we expect independence between the instantaneous amplitude derivatives and the instantaneous frequency as given by:

$$\langle \boldsymbol{a},\, \boldsymbol{\zeta} \rangle \neq 0. \tag{10}$$

Clearly, under (10) we have a set of two independent equations in two unknowns as given by (6) and (8).

B. Smooth Motion

Even if (7) and (9) provide two equations in two variables, we have an aperture-like problem in that we cannot estimate motions that are orthogonal to the IF and IA. Assuming smooth motions, we follow [9] by introducing a penalty function that requires small first-order derivatives of the optical flow [3]. Thus, we use the following penalty function based on the ℓ-2 norm:

$$E_{priori} = \sum_{\mathbf{z}} |\nabla u(\mathbf{z})|^2 + |\nabla v(\mathbf{z})|^2 = \|\boldsymbol{\zeta}\|_2^2. \tag{11}$$

The overall optimization problem is to compute the optical flow functions that minimize $E_{Global} = \int E(u(\mathbf{z}), v(\mathbf{z}), \mathbf{z}, \nabla u(\mathbf{z}), \nabla v(\mathbf{z}))d\mathbf{z}$ with (7) and (11) given by

$$E(u(\mathbf{z}), v(\mathbf{z}), \mathbf{z}, \nabla u(\mathbf{z}), \nabla v(\mathbf{z})) = E_{priori} + \lambda E_{FM} + \beta E_{AM}, \tag{12}$$

where λ and β are regularization parameters.

C. Solution using Total Variation methods

The result from (12) is similar to the general energy penalty given in [9] for a two-dimensional flow field. However, here the flow field comes from the AM-FM model (described in subsection III-A).

We propose to replace the quadratic penalties [3] in (11) with a robust version, e.g. $E_{priori} = \sum_{\mathbf{z}} |\nabla u(\mathbf{z})| + |\nabla v(\mathbf{z})| = \|\boldsymbol{\zeta}\|_1$, which is known to better preserve discontinuities. Thus, the problem described by (12) is equivalent to the vector-valued ℓ^2 Total Variation (TV) optimization:

$$T(\boldsymbol{\zeta}) \;=\; \frac{1}{2}\|A \cdot \boldsymbol{\zeta} - \gamma(\mathbf{z})\|_2^2 + \lambda \|\nabla \boldsymbol{\zeta}\|_1, \tag{13}$$

where $A = diag(\nabla\varphi(\mathbf{z}))$. We use the Iteratively Reweighted Norm for total variation (IRN-TV) algorithm [16] to solve the problem because of its simplicity and good computational performance.

Given $T(\boldsymbol{\xi}) = \frac{1}{2}\|A\boldsymbol{\xi} - \mathbf{b}\|_2^2 + \lambda R(\boldsymbol{\xi})$, where $\boldsymbol{\xi}$, the dataset to be restored, represents a vector-valued dataset with L elements per entry, $R(\boldsymbol{\xi}) = \|\sqrt{\sum_1^L (D_x\boldsymbol{\xi}_n)^2 + (D_y\boldsymbol{\xi}_n)^2}\|_1$ is the discrete version of $\|\nabla\boldsymbol{\xi}\|_1$, with D_x and D_y representing the horizontal and vertical discrete derivative operators respectively, A is the forward operator, $\mathbf{b}$ is the observed noisy data and λ is a weighting factor controlling the relative importance of the data fidelity and regularization terms. The main idea is to express the regularization term by the quadratic approximation $Q_R^{(k)}(\boldsymbol{\xi}) = \frac{1}{2}\|W_R^{(k)^{1/2}} D\boldsymbol{\xi}\|_2^2$, where $W_R^{(k)} = I_{2L} \otimes \Omega_R^{(k)}$, $D = [D_x^T D_y^T]^T$, $\Omega_R^{(k)} = diag\left(\left(\sum_1^L (D_x\boldsymbol{\xi}^{(k)})^2 + (D_y\boldsymbol{\xi}^{(k)})^2\right)^{-0.5}\right)$, I_N is a $N \times N$ identity matrix, and $\otimes$ is the Kronecker product. The resulting iterations can be expressed in the form of the standard iteratively reweighted least squares (IRLS) problem:

$$T^{(k)}(\mathbf{u}) = \frac{1}{2}\left\| \begin{bmatrix} 1 & 0 \\ 0 & W_R^{(k)} \end{bmatrix}^{1/2} \begin{bmatrix} A \\ \sqrt{\lambda} D \end{bmatrix} \boldsymbol{\xi} - \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \right\|_2^2.$$

For a given current solution $\boldsymbol{\xi}^{(k)}$, $W_R^{(k)}$ can be computed, and the threshold τ may be automatically adapted to the input image to avoid numerical instability [16]. Finally, the resulting IRN algorithm has to iteratively solve the linear system $\left(A^T A + \lambda D^T W_R^{(k)} D\right) \boldsymbol{\xi}^{(k+1)} = A^T \mathbf{b}$.

IV. Results and Discussion

We have test this first approach using synthetic videos using 50 video frames with 512×512 pixels in each frame. We first define the reference image using $I(\mathbf{z}) = \cos\varphi(\mathbf{z})$, where $\varphi(\mathbf{z}) = \varphi(x, y) = 2\pi\left[\alpha_1 x + \beta_1 \frac{x^2}{2} + \alpha_2 y + \beta_2 \frac{y^2}{2}\right]$ and $\nabla\varphi(\mathbf{z}) = \nabla\varphi(x, y) = 2\pi\,(\alpha_1 + \beta_1 x,\; \alpha_2 + \beta_2 y)$, with the parameters $\alpha_1, \beta_1, \alpha_2$ and β_2 set to produce the instantaneous frequencies values in the range of $\varphi_x \in [0.10, 0.15]$ and $\varphi_y \in [0.15, -0.13]$ in the normalized frequency range $[0, 1]$. The relatively low frequency values give smoothness to the video and make it more difficult to produce motion vector estimates from higher-frequency channel filters. We consider three cases: $(u, v) = (2, -0.5)$, $(u, v) = (1, 0)$, and $(u, v) = (0, 0.7)$.

We present the results in terms of accuracy in the motion estimation in the x and y directions given by the vectors u and v, respectively. Also, we analyze the density in the estimation, which represents the percentage of pixels with a motion estimation vector computed. This number is very important in applications like in the medical applications,

TABLE I
RESULTS IN TERMS OF THE MEAN-SQUARED ERROR (MSE) FOR THE VECTORS (u, v) AND THE DENSITY OF ESTIMATION (%) BY VIDEO. WE COMPARE THE PROPOSED METHOD VERSUS THOSE DESCRIBED IN SECTION II.

Method	$(u,v) = (2, -0.5)$			$(u,v) = (1, 0)$			$(u,v) = (0, 0.7)$		
MSE	u	v	%	u	v	%	u	v	%
Proposed 2D AM-FM	1.29	0.69	1.00	0.11	0.10	1.00	0.12	0.15	1.00
2D IF-based [2]	2.29	1.29	1.00	0.17	0.20	1.00	0.24	0.30	1.00
3D AM-FM	10.95	8.37	0.85	2.58	1.29	0.85	0.07	1.01	0.85
Horn-Schunck	4.16	1.25	1.00	1.04	0.20	1.00	0.20	0.49	1.00
Lucas-Kanade	11.22	1.01	0.34	2.78	0.17	0.32	0.08	0.87	0.31
Gunnar Farnebäck	7.86	0.64	1.00	2.14	0.05	1.00	0.02	0.86	1.00
TVL1	23.74	0.76	1.00	3.26	0.16	1.00	0.11	1.09	1.00

where every single voxel must be tracked (for example [7]). We have used the available functions in Matlab and OpenCV, library with CPU and GPU, to compare the results. From this, we obtain the results to discuss about the efficiency of the all methods used. For the accuracy errors, we compute the mean-squared error (MSE) for the vectors (u, v) in Table I.

We can see how the proposed method not only produce the motion estimation vectors at every single voxel (density of 100%) but also it produce the best results. Since the AM-FM estimation methods do not produce perfect results [17], those variations are used by the TV approach to have a better estimation. In terms of the motion estimation functions found in open source and commercial software, we have noted that the CPU based functions are more accurate for synthetic videos.

V. CONCLUSIONS AND FUTURE WORK

We have presented an improved perception-based motion estimation method based on the use of the 2D AM-FM demodulation methods over two consecutive frames. We have also provided a solution to the problem based on the use of total variation algorithms. We have compared the proposed solution versus previous approaches, such as 2D IF-based methods and 3D AM-FM-based methods, and functions available in open source and commercial software. In the synthetic examples, the proposed approach gave significantly better motion estimation results with a full density estimation at every single frame. Future research will extend the method to the use of multiscale filterbanks and to model the influence of external factors such as not constant illumination. This method will be tested in standard video sequences and ultrasound videos.

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