2D Instantaneous Frequency-based Method for Motion Estimation using Total Variation

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***Abstract*—We present a first approach to a new method to compute the motion estimation in digital videos using the two-dimensional instantaneous frequency information computed using amplitude-modulation frequency-modulation (AM-FM) methods. The optical flow vectors are computed using an iteratively reweighted norm for total variation (IRN-TV) algorithm. We compare the proposed method using synthetic videos versus a previous three-dimensional AM-FM based method and available motion estimation methods such as a phase-based, Horn-Schunck and the Lucas-Kanade methods. The results are promising producing a full density estimation with more accurate results than the other methods.**

***Index Terms*—motion estimation, optical flow, amplitude-modulation frequency-modulation**

I. Introduction

There is strong interest in the development of perception-based motion estimation methods that are based on multi-scale amplitude-modulation frequency-modulation (AM-FM) methods [1]. The standard use of perception-based methods for motion estimation methods rely on the use of 3D Gabor filter banks that require a large number of video frames as discussed in [1]. In this paper, we present a new approach that uses a multi-scale AM-FM method to estimate optical flow motion using the instantaneous frequency information from two video frames only.

Our motivation for the new perception-based approach comes from the need to maintain compatibility with the majority of optical flow motion methods that are based on a small number of video frames [2]. Recent work on the use of AM-FM methods in biomedical imaging was motivated by the research described in [3] that led to improved methods described in [4], [5]. Earlier research on phase-based methods was reported in [6]. However, as shown in [7], these methods require smooth variations that have a very difficult time with ultrasound videos that are dominated by speckle noise. For speckled ultrasound videos, there are no perception-based methods that can produce motion estimation estimates with a 100% density [8]. A large number of non AM-FM based methods is recently reported in [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22].

In this paper, we propose a first approach to a motion estimation based on AM-FM that goes beyond the 3D AM-FM methodology described in [5], [23], [1]. This method only requires two video frames: $I(x, y, t)$ and $I(x, y, t+1)$. We use the term local method to refer to the fact that there is no requirement to have a collection of video frames for estimating time-derivatives or requiring 3D convolutions. Furthermore, to allow computations from sparse estimates that are sensitive to object boundaries, we use a total variation formulation of our approach. In this first approach of our new method, we present comparative results on synthetic examples where we show that the proposed approach provides better estimates than many other standard methods.

In the rest of the paper, we provide background on related work in section II. Next, we present the methods proposed in section III. We test the proposed method using synthetic videos to present the first results of this new method in section IV. Finally, we present the conclusions and future work in section V.

II. Background

We represent a digital video $I(\mathbf{z}, t)$, where $\mathbf{z} = (x, y)$, in terms of a collection of AM-FM components as given by:

$$I(\mathbf{z}, t) = \sum_{n=1}^{n=M} a_n(\mathbf{z}, t) \cos(\varphi_n(\mathbf{z}, t)), \tag{1}$$

where $n = 1, 2, \ldots, M$ denotes the different AM-FM components, $a_n(\mathbf{z}, t)\cos(\varphi_n(\mathbf{z}, t))$ denotes the n-th AM-FM component, a_n denotes the n-th instantaneous amplitude (IA) component, and φ_n denotes the n-th instantaneous phase (IP) component. The instantaneous frequency (IF) is defined in terms of the gradient of the instantaneous phase: $\nabla\varphi_n$. Example applications of the use of AM-FM models in medical imaging are described in [24], [25], [26], [27], [28].

For the 3D AM-FM motion estimation method [23], we consider a single AM-FM component given by $I(\mathbf{z}, t) = a(\mathbf{z}, t)\exp(j\varphi(\mathbf{z}, t))$. Next, we consider the Optical Flow Constraint Equation (OFCE, [29]) given by

$$\langle \nabla I, \boldsymbol{\zeta} \rangle + \frac{\partial}{\partial t} I = 0, \tag{2}$$

where $\boldsymbol{\zeta} = (u, v)$, $\langle . \rangle$ represents inner product, and we apply it to $I(\mathbf{z}, t)$. Then, by inspecting the real part of the equation, we obtain the AM constraint given by

$$\langle \nabla a, \boldsymbol{\zeta} \rangle + \frac{\partial}{\partial t} a = 0, \tag{3}$$

and similarly, by examining the imaginary part only, we obtain the FM constraint given by

$$\langle \nabla\varphi, \boldsymbol{\zeta}\rangle + \frac{\partial}{\partial t}\varphi = 0. \tag{4}$$

The advantage of (3)-(4) is the use of two equations per voxel. The final motion estimation vectors $u(x, y)$ and $v(x, y)$ are computed using an iterative method based on finite difference approximations [30]. In the next section, we present our first approach to a new method for motion estimation using 2D AM-FM methods to computed the features of two consecutive frames by the time. Also, the solution is computed using total variation methods.

III. Methods

We will first develop a method for motion estimation based on a single AM-FM component as described in subsection III-A. Our goal is to compute the motion estimation vectors by processing only two consecutive frames at a time using a 2D AM-FM demodulation method. Next, in subsection III-B, we extend our approach by defining a penalty function that can be used with smooth video pairs with a small number of feature points. In subsection III-C, we propose a total variation method that is used for solving the optimization problem associated with motion estimation.

A. Basic Model for a Single AM-FM Component

We want to solve the motion estimation problem using two consecutive frames from a digital video $I(\mathbf{z}, t)$. Let $I_1(\mathbf{z}) = I(\mathbf{z}, t)$ denote the image intensity function at time t. Then, let $I_2(\mathbf{z}) = I(\mathbf{z}, t+\triangle t)$ denote the image intensity at time $t+\triangle t$. We formulate the optical flow motion estimation problem for each pixel $\mathbf{z} = (x, y)$ as one of determining the optical flow functions $\boldsymbol{\zeta}(\mathbf{z}) = (u(\mathbf{z}), v(\mathbf{z})) = (u(x, y), v(x, y))$ from I_1 and I_2.

We assume constancy in image intensity as given by $I(\mathbf{z}, t)$ to $I(\mathbf{z}, t + 1)$. Furthermore, we consider the basic model for a single AM-FM component given by:

$$I(\mathbf{z}, t) = a(\mathbf{z}, t) \exp[j\varphi(\mathbf{z}, t)]. \tag{5}$$

Then, for two frames, we have:

$$\begin{aligned} I(\mathbf{z}, t + 1) &= I(\mathbf{z} + \boldsymbol{\zeta}, t) \\ &= a(\mathbf{z} + \boldsymbol{\zeta}, t) \exp[j\varphi(\mathbf{z} + \boldsymbol{\zeta}, t)]. \end{aligned}$$

Based on (2) and given that $I_k(\mathbf{z}) = a_k(\mathbf{z}) \cos\varphi_k(\mathbf{z})$, with $k \in \{1, 2\}$, we have that:

$$\begin{aligned} I_2(\mathbf{z}) &= a_2(\mathbf{z}) \cos\varphi_2(\mathbf{z}) \\ &= I_1(\mathbf{z} + \boldsymbol{\zeta}) \\ &= a_1(\mathbf{z} + \boldsymbol{\zeta}) \cos\varphi_1(\mathbf{z} + \boldsymbol{\zeta}). \end{aligned} \tag{6}$$

Then, similar to the derivation in [31], we extract the FM image by dividing the image output by the IA as given by: $\bar{I}(\mathbf{z}, t) = (a(\mathbf{z}, t) \exp[j\varphi_1(\mathbf{z}, t)])/a(\mathbf{z}, t) = \exp[j\varphi_1(\mathbf{z}, t)]$. We next consider the simplest case given by $\varphi(\mathbf{z}) = \langle \boldsymbol{\omega}, \mathbf{z}\rangle + c$, where c is a phase constant, $\boldsymbol{\omega} = (\omega_1(\mathbf{z}), \omega_2(\mathbf{z})) = (\omega_1(x, y), \omega_2(x, y))$, $\langle . \rangle$ represents inner product, and $\varphi_1(\mathbf{z}) = \varphi(\mathbf{z})$. In this case, we get:

$$\begin{aligned} \frac{\bar{I}_2(\mathbf{z})}{\bar{I}_1(\mathbf{z})} &= \frac{I(\mathbf{z}, t + \triangle t)}{I(\mathbf{z}, t)} \\ &= \exp[j(\varphi(\mathbf{z} + \boldsymbol{\zeta}, t) - \varphi(\mathbf{z}, t))] \\ &= \exp[j\langle \boldsymbol{\omega}, \boldsymbol{\zeta}\rangle]. \end{aligned} \tag{7}$$

More generally, by applying (7) to (2) we have the basic linear-phase model approximation given by:

$$\gamma(\mathbf{z}) = \langle \boldsymbol{\phi}, \boldsymbol{\zeta}\rangle = \texttt{Arg}\left(\frac{\bar{I}_2(\mathbf{z})}{\bar{I}_1(\mathbf{z})}\right), \tag{8}$$

where $\boldsymbol{\phi} = \nabla\varphi(\mathbf{z}) = (\varphi_x(\mathbf{z}), \varphi_y(\mathbf{z}))$ are the x and y components of the instantaneous frequency (IF) vectors. Over the entire image, we form the error in solving for (8) using an energy function given by

$$E_{FM} = \sum_{\mathbf{z}} \left[\langle \boldsymbol{\phi}, \boldsymbol{\zeta}\rangle - \gamma(\mathbf{z})\right]^2. \tag{9}$$

B. Penalty function

Since (9) provides a single equation in two variables, we have an aperture-like problem in that we cannot estimate motions that are orthogonal to the IF. Assuming smooth motions, we follow [32] by introducing a penalty function that requires small first-order derivatives of the optical flow [2]. Thus, based on this prior assumption, we introduce the following penalty function based on the ℓ-2 norm:

$$E_{priori} = \sum_{\mathbf{z}} |\nabla u(\mathbf{z})|^2 + |\nabla v(\mathbf{z})|^2 = \|\boldsymbol{\zeta}\|_2^2. \tag{10}$$

Then, the overall optimization problem is to compute the optical flow functions that minimize $E_{Global} = \int E(u(\mathbf{z}), v(\mathbf{z}), \mathbf{z}, \nabla u(\mathbf{z}), \nabla v(\mathbf{z}))d\mathbf{z}$ with (9) and (10) given by

$$E(u(\mathbf{z}), v(\mathbf{z}), \mathbf{z}, \nabla u(\mathbf{z}), \nabla v(\mathbf{z})) = E_{priori} + \lambda E_{FM}, \tag{11}$$

where λ is a regularization parameter.

C. Solution using Total Variation

The main result of the previous sections are similar to the general energy penalty given in [32] for a two-dimensional flow field. However, here the flow field comes from the FM part of the AM-FM model (described in subsection III-A).

As it has been noticed before [2], it is convenient to replace the quadratic penalties in (10) with a robust version, e.g. $E_{priori} = \sum_{\mathbf{z}} |\nabla u(\mathbf{z})| + |\nabla v(\mathbf{z})| = \|\boldsymbol{\zeta}\|_1$, which is known [13] to better preserve discontinuities. Then, the problem described by (11) is equivalent to the vector-valued ℓ^2 Total Variation (TV) optimization:

$$T(\boldsymbol{\zeta}) = \frac{1}{2}\|A \cdot \boldsymbol{\zeta} - \gamma(\mathbf{z})\|_2^2 + \lambda\|\nabla\boldsymbol{\zeta}\|_1, \tag{12}$$

where $A = diag(\nabla\varphi(\mathbf{z}))$. We note that there is a long list of numerical algorithms (for instance, see [33]) that can solve (12). Here we choose to use the Iteratively Reweighted Norm for total variation (IRN-TV) algorithm [34], [35] due to its simplicity and good computational performance.

Given $T(\boldsymbol{\xi}) = \frac{1}{2}\|A\boldsymbol{\xi} - \mathbf{b}\|_2^2 + \lambda R(\boldsymbol{\xi})$, where $\boldsymbol{\xi}$, the dataset to be restored, represents a vector-valued dataset with L elements per entry, $R(\boldsymbol{\xi}) = \|\sqrt{\sum_1^L (D_x\boldsymbol{\xi}_n)^2 + (D_y\boldsymbol{\xi}_n)^2}\|_1$ is the discrete version of $\|\nabla\boldsymbol{\xi}\|_1$, with D_x and D_y representing the horizontal and vertical discrete derivative operators respectively, A is the forward operator, $\mathbf{b}$ is the observed noisy data and λ is a weighting factor controlling the relative importance of the data fidelity and regularization terms.

The key idea [34] is to express the regularization term by the quadratic approximation $Q_R^{(k)}(\boldsymbol{\xi}) = \frac{1}{2}\|{W_R^{(k)}}^{1/2} D\boldsymbol{\xi}\|_2^2$, where $W_R^{(k)} = I_{2L} \otimes \Omega_R^{(k)}$, $D = [D_x^T D_y^T]^T$, $\Omega_R^{(k)} = diag\left(\left(\sum_1^L (D_x\boldsymbol{\xi}^{(k)})^2 + (D_y\boldsymbol{\xi}^{(k)})^2\right)^{-0.5}\right)$, I_N is a $N \times N$ identity matrix, and $\otimes$ is the Kronecker product. The resulting iterations can be expressed in the form of the standard iteratively reweighted least squares (IRLS) problem:

$$T^{(k)}(\mathbf{u}) = \frac{1}{2}\left\|\begin{bmatrix} 1 & 0 \\ 0 & W_R^{(k)} \end{bmatrix}^{1/2}\begin{bmatrix} A \\ \sqrt{\lambda}D \end{bmatrix}\boldsymbol{\xi} - \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix}\right\|_2^2 \quad (13)$$

For a given current solution $\boldsymbol{\xi}^{(k)}$, the weighting matrix $W_R^{(k)}$ can be easily computed, and the threshold τ may be automatically adapted to the input image to avoid numerical instability [34]. Finally, the resulting IRN algorithm has to iteratively solve the linear system

$$\left(A^T A + \lambda D^T W_R^{(k)} D\right)\boldsymbol{\xi}^{(k+1)} = A^T\mathbf{b}. \quad (14)$$

IV. Results and Discussion

We present results on a synthetic video example using 50 video frames with 512×512 pixels in each frame. We first define the reference image using

$$I(\mathbf{z}) = \cos\varphi(\mathbf{z}),$$

where

$$\varphi(\mathbf{z}) = \varphi(x,y) = 2\pi\left[\alpha_1 x + \beta_1\frac{x^2}{2} + \alpha_2 y + \beta_2\frac{y^2}{2}\right],$$
$$\nabla\varphi(\mathbf{z}) = \nabla\varphi(x,y) = 2\pi\left(\alpha_1 + \beta_1 x,\ \alpha_2 + \beta_2 y\right),$$

with the parameters $\alpha_1, \beta_1, \alpha_2$ and β_2 set to produce the instantaneous frequencies values in the range of $\varphi_x \in [0.10, 0.15]$ and $\varphi_y \in [0.15, -0.13]$ in the normalized frequency range $[0, 1]$. The relatively low frequency values give smoothness to the video and make it more difficult to produce motion vector estimates from higher-frequency channel filters.

We consider three cases:

- $(u,v) = (2, -0.5)$,
- $(u,v) = (1, 0)$, and
- $(u,v) = (0, 0.7)$.

In Fig. 1, we show the frames 1, 25 and 50 for each of the three videos. In Table I, we summarize the results after computing the optical flow vectors (u,v) using the proposed method versus the 3D AM-FM based [31], the phase-based approach by [36], Horn-Schunck (H-S, [32]) and the Lucas-Kanade (L-K, [37]) methods[1]. The results are presented in terms of the mean-squared error (MSE) for the vectors (u,v) for the 50 frames of the video and the density of the estimation (e.g., the percentage of the pixels per frame where the method was able to computed the optical flow vectors). For pixels where the motion produced $(u,v) = (0,0)$ or not a number, we consider that the method has failed.

We can observe in Fig. 1 the difficulty for finding feature points to compute the optical flow methods. From Table I, we can see that only the proposed method and Horn-Schunck (H-S, [32]) are able to compute a full density motion estimation. The 3D AM-FM based method produced the result $(0,0)$ for some voxels, reducing the percentage of motion estimation vectores computed. Note that [36] was not able to compute the optical flow for these videos.

In terms of accuracy, the proposed method produced better results than the 3D AM-FM based method for all the videos except for the case of $u = 0$ in video 3 (MSE=0.24 versus MSE=0.07). Compared to the Horn-Schunck method, the proposed approach gave better results with much lower MSE in most cases, except in two cases, where the Horn-Schunck method produced slightly better results. The Lucas and Kanade method had an unacceptable density that never exceeded 35% as opposed to the 100% density of the proposed method. Within the 35% reduced density, Lucas and Kanade would sometimes give slightly better results, but often gave significantly worse results.

V. Conclusions and Future Work

We have presented a perception-based motion estimation method based on the use of the 2D instantaneous frequency and two video frames. We also provided an optimized approach based on the use of total variation to allow for robust solutions. We also provided comparative results to other phase-based and standard methods. Overall, in our synthetic examples, the proposed approach gave significantly better estimates while estimating motion everywhere. Future research will extend the method to the use of the 2D instantaneous amplitude and will also be extended to combine estimates from several scales. The approach will be tested on standard video sequences as well as simulated ultrasound videos.

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[1]Computed using the Matlab OpticalFlow System object™.

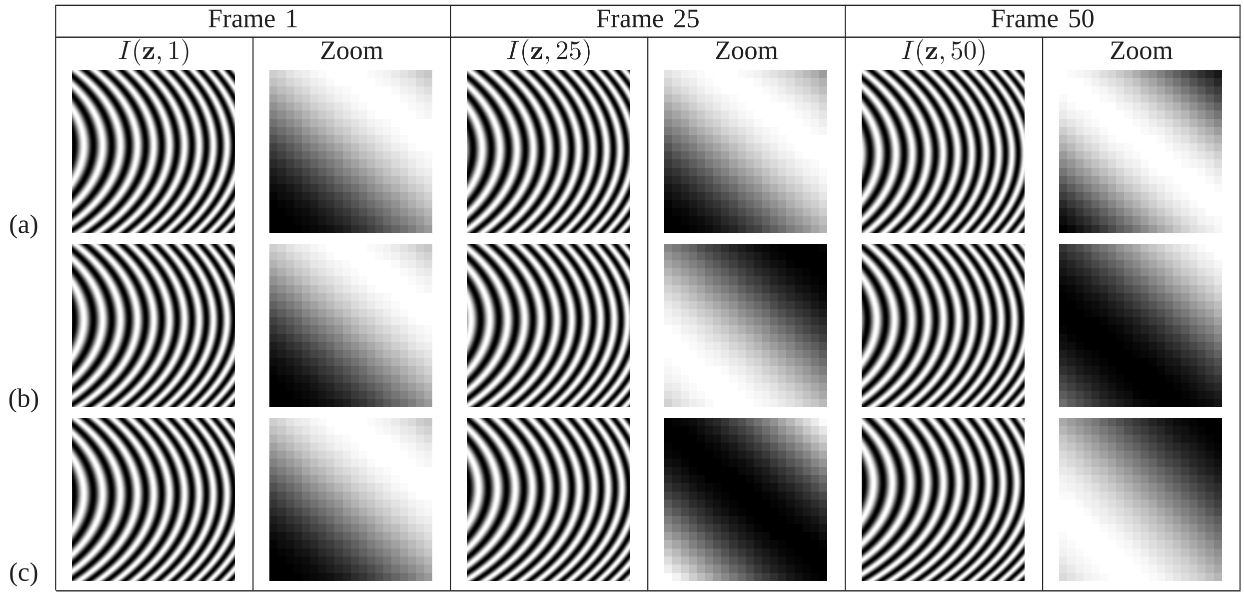

Fig. 1. Synthetic videos used for the tests. Videos with $(u, v) = (2, -0.5)$, $(u, v) = (1, 0)$, and $(u, v) = (0, 0.7)$ in rows (a), (b), and (c), respectively. We show the frames 1, 25 and 25 for each video. Also, we zoom on the top left corner of each frame (rows and columns from 1 to 20) to show more details about the changes in the pixels.

TABLE I

SUMMARY OF THE RESULTS IN TERMS OF THE MEAN-SQUARED ERROR (MSE) FOR THE VECTORS (u, v) AND THE DENSITY OF ESTIMATION BY VIDEO. WE COMPARE THE PROPOSED METHOD VERSUS THE 3D AM-FM BASED [31], THE PHASE-BASED APPROACH BY [36], HORN-SCHUNCK (H-S, [32]) AND THE LUCAS-KANADE (L-K, [37]) METHODS.

Method	$(u, v) = (2, -0.5)$			$(u, v) = (1, 0)$			$(u, v) = (0, 0.7)$		
MSE	u	v	%	u	v	%	u	v	%
Proposed	2.29	1.29	1.00	0.17	0.20	1.00	0.24	0.30	1.00
3D AM-FM	10.95	8.37	0.85	2.58	1.29	0.85	0.07	1.01	0.85
Phase-based method from [36]	-	-	0.00	-	-	0.00	-	-	0.00
H-S	4.16	1.25	1.00	1.04	0.20	1.00	0.20	0.49	1.00
L-K	11.22	1.01	0.34	2.78	0.17	0.32	0.08	0.87	0.31

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